

Homework 7

OR 41

- (20 points) Assume that in the Monty Hall problem there are n doors, but still only one prize. Analyze the chance of winning with the switching strategy.
- A gambling book presents the following strategy for roulette. It recommends that
 - the gambler bet 1 \$ on red. If he/she wins, then quit.
 - If he/she loses, then bet 2 \$ on red. Then quit, no matter whether he/she won, or lost.

Let X be the profit of the gambler. Find out for which k values is $P(X = k) > 0$? Find $E(X)$. Is it better than the expected winnings of just one bet on red? (15 points) Recall that the probability of getting a red in roulette is $18/38$.

Consider the following generalization. Fix a number ℓ ; $\ell + 1$ will be maximum number of betting rounds the gambler will make. That is,

- the gambler bets 1 \$ on red. If he/she wins, then quit.
- If he/she loses, then bets 2 \$ on red. If he/she wins, then quit.
- etc.
- If he/she loses, then bets $2^{\ell-1}$ \$ on red. If he/she wins, then quit.
- If he/she loses, then bets 2^ℓ \$ on red. Then quit, no matter whether he/she won, or lost.

Call this strategy the ℓ -strategy. Find the expected profit in the ℓ -strategy, if the probability of winning is p .

For what value of p does the expected profit increase, when ℓ gets larger? For what values does it decrease?

(40 points)

- The game of craps, played with two dice, is one of America's fastest and most popular gambling games. The most common bet is a Pass Line bet. Here are the rules:

Only the totals for the two dice count. The player throws the dice and wins at once if the total for the first throw is 7 or 11, and loses at once if it is a 2, 3, or 12. Any other throw is called his "point". If the first throw is a point, the player throws the dice repeatedly until he either wins by throwing his point again or loses by throwing a 7.

What is the player's chance to win?

(Hint: Condition on the first roll and use the Law of Total Probability). (30 points)

4. A total 46 percent of the voters in a certain city classify themselves as Independents, whereas 30 percent classify themselves as Liberals, and 24 percent as Conservatives. In a recent local election, 35 percent of the Independents, 62 percent of the Liberals, and 58 percent of the Conservatives voted. A voter is chosen at random.

- (a) Given that this person voted in the local election, what is the probability that he or she is an Independent? (10 points)
- (b) What fraction of the voters participated in the election? (Convince yourself this is the same question as: What is the probability a randomly selected voter participated in the election?) (10 points)

5. Two hundred \$2 tickets are sold for a raffle that has 5 prizes: 1 – \$100, 1 – \$50, 1 – \$25, 2 – \$10. Let X = Value of a random ticket.

- What are the values k , so that $P(X = k) > 0$? For these values, find $P(X = k)$.
- What is the expected value of X ?

(10 points total)

6. An insurance representative has appointments with 4 prospective clients tomorrow. From past experience, she knows that the probability of making a sale on any appointment is 0.20, and each appointment is independent of each other.

- (a) What is the probability that she will sell a policy to at least 3 of the 4 prospective clients?
- (b) What is the expected number of policies that she will sell tomorrow?

(15 points total)

7. Ross 3.12 (15 points)

8. Ross 3.15 Part(a) only. (10 points)

9. Ross 3.16

You don't need to calculate the standard deviation. However, please answer these additional questions: Which gives the lawyer the higher expected value, the fixed fee or the contingency fee? Does that mean it must be the best strategy for the lawyer? Explain. (15 points total)

10. Solve the following linear program using the graphical method. Shade the feasible region, and find the optimal solution and the optimal objective value.

$$\begin{array}{ll}
\text{minimize} & 10x + 4y \\
\text{subject to} & 3x + 4y \leq 12 \\
& 6x + 3y \leq 15 \\
& x \geq 1, y \geq 0
\end{array}$$

(10 points)

11. Susie Sewingqueen sells dress patterns at her fabric store, and stocks both the Butterick and Simplicity brands. She is trying to decide how many of patterns from each company to stock for the coming summer season. Her profits are \$2.50 per Butterick pattern and \$3.00 per Simplicity pattern. Simplicity patterns are a little thicker and require 9 mm of storage space in the pattern drawers, while Butterick patterns only require 6 mm of space. Susie has a total of 72 centimeters of storage space available to hold the patterns. Susie must purchase all her patterns at the beginning of the season, and she has only \$300 available for those purchases. Each Butterick pattern costs her \$4, while Simplicities cost her \$3. She anticipates selling every pattern that she stocks.
- (a) Formulate a linear program which will help Susie maximize her profits during the summer season. Please define your decision variables and label your constraints. (10 points)
- (b) Use the graphical method to find an optimal solution to your LP. Please shade in the feasible region. How many of each type of pattern should Susie stock? How much profit will she make? (10 points)

The following questions ask you to modify your LP. In answering them, please be sure to define any necessary new variable(s) and list any change(s) to the constraints and/or objective function. **You don't need to rewrite the entire LP unless you feel it will be easier to do that than to merely list the changes.**

- (a) Susie is also considering stocking Vogue patterns. Each Vogue pattern will earn her a profit of \$3.25, while occupying 11 mm of space and costing her \$2.50. How will this change your formulation in part (a)? (3 points)
- (b) How does your formulation in part (c) change if Susie wants to stock at least as many Simplicity patterns as Vogue patterns? (3 points)
- (c) How does your formulation in part (c) change if Susie wants no more than 40% of her stock to come from companies other than Simplicity? (3 points)
12. Consider the post-office workscheduling problem as seen in class. Suppose that an employee makes 100 dollars a day. However, he/she may be asked to work overtime on the 6th day after his/her regular 5 working days; on this day, he/she will be payed 130 dollars.

Modify the LP to reflect this change. For simplicity assume that if an employee is asked to work overtime, then he/she will be asked *every* week. (30 points)